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Origami and Compliant Mechanisms

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Abstract. Origami is the art of folding paper. In the context of engineering, orimimetics is the application of folding to solve problems. There is a branch of origami called "action origami" where origami models in their final-folded state exhibit motion. This motion can be modelled with the pseudo-rigid-body model since they are compliant mechanisms. These mechanisms, when having a flat initial state and motion emerging out of the fabrication plane, are classified as lamina emergent mechanisms (LEMs). In this paper, four flat folding paper mechanisms are presented with their corresponding kinematic and graph models. Principles from graph theory are used to abstract the mechanisms to show them as coupled, or inter-connected, mechanisms. It is anticipated that this work lays a foundation for exploring methods for LEM synthesis.

Keywords. Compliant Mechanisms, Lamina Emergent Mechanisms, Origami, orimimetics

1 Introduction

Origami is the art of folding paper where *ori-* means fold and *-kami* means paper. The scope and complexity of origami has exploded in the last twenty years creating many different schools of thought (Demaine et al., 2010). Recently in the fields of science, mathematics and engineering, origami has been used to solve complex problems such as airbag folding, shock absorption (crash box), and deployable telescopic lenses (Cromvik, 2007; Ma and You, 2010; Heller, 2003). Origami design is governed by mathematical laws, which if better understood, could be applied in engineering. Specifically, the authors believe that origami can advance synthesis techniques for compliant mechanisms. This paper examines the potential that origami has to provide such solutions and new tools for mechanism design.

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Traditionally origami has been static and representational, however, action origami focuses on origami models that have some novel motion. Figure 1 shows two examples, where one is a static origami structure and the other is an action origami mechanism called a flasher hat; it is shown in its fabricated and deployed forms. Another branch of origami, rigidly foldable origami, focuses on motion (Tachi, 2006; Balkcom and Mason, 2008; Hull, 1994; Watanabe and Kawaguchi, 2006). In this class of origami the creases often act as joints and the faces act as links with bending stress only occurring at the creases.

Fig. 1. Example of an origami structure (left) and flasher hat mechanism in its fabricated (middle) and deployed form (right).

The process of folding origami has been examined using kinematic theory (Dai and Jones, 2002; Balkcom and Mason, 2008; Tachi, 2006; Buchner, 2003). This paper focuses on the motion of a finished origami and not the states in between flat and final states.

Since origami relies on the deflection of flexible materials it is a compliant mechanism. The origami mechanisms examined herein are all flat folding in their final folded state, and so they are part of subgroup of compliant mechanisms called lamina emergent mechanisms (LEMs). Lamina emergent mechanisms are compliant mechanisms made from planar materials (lamina) with motion that emerges out of the fabrication plane.

The connection between origami and mechanisms is also seen by drawing upon principles from graph theory to depict the origami mechanisms via simple, planar connected graphs. The purpose of this is to show how the folds and facets interact with each other in their motion. Graphs allow for improved understanding of the interaction between motion and structure of origami, and may help in understanding how to predict complex motion.

Origami's rich history and research in design optimisation can provide an orimimetic perspective to LEM design to develop products requiring complex motion from compact mechanisms.

1.1 Objective

Orimimetic design refers to the application of the concepts of folding to improve design. It can provide alternative ways to achieve a particular range of motion; compliant mechanisms have a similar aim. In addition, origami is achieved from the manipulation of a planar material which motivates its examination as a lamina emergent mechanism. By showing the relation between origami and compliant mechanisms it is anticipated that the literature for origami design can be applied to compliant mechanism design, specifically in the area of LEMs.

2 Nomenclature

It is helpful to establish terminology to facilitate discussion. A few terms, and their use, are provided below.

2.1 Origami

Origami. Origami is defined as the art of folding paper, and in the context of engineering, it is the use of folding to solve mechanical problems (Demaine and O'Rourke, 2007).

Kirigami. Kirigami is defined as the art of folding and cutting paper (Hart, 2007). An example of kirigami is shown in Figure 2 with the side view showing how creases are made such that the house pops out of the paper.

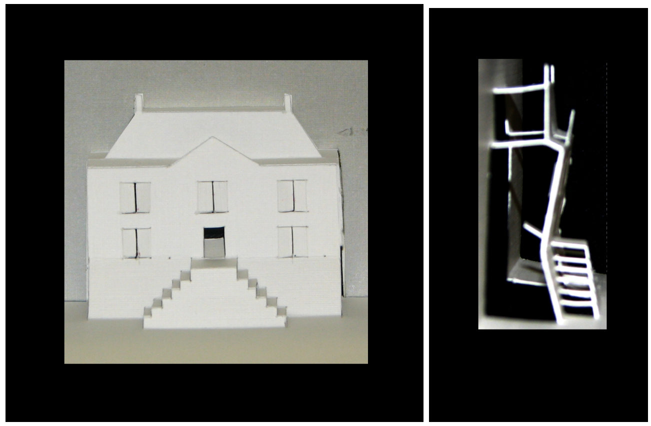

Fig. 2. A kirigami house made in cardstock.

Hinge creases. In traditional origami hinge creases define the boundaries between flaps. In the context of mechanisms they are creases along which lies an interface of two planar faces and it is the axis about which both facets rotate (Demaine and O'Rourke, 2007).

Construction creases. Creases used to create references in the construction of the mechanism, but are not directly used to create motion, are referred to as construction creases. In some cases, they coincide with hinge creases (Demaine and O'Rourke, 2007).

Structural creases. Structural creases are used to define the shape of flaps; they can be hinge creases as well. These creases are not needed in most mechanisms and they are not feasible for many materials. They may/can be substituted or eliminated in various ways (Demaine and O'Rourke, 2007).

Crease vs. fold. A fold is an action and a crease is the product of that action.

2.2 Mechanisms

Compliant Mechanism. Mechanisms which transfer or transform motion, force, or energy at least in part through the deflection of flexible members are compliant mechanisms (Howell, 2001).

Lamina Emergent Mechanism. Lamina emergent mechanisms (LEMs) are a subset of compliant mechanisms made from planar materials (lamina) with motion that emerges out of the fabrication plane (Jacobsen, 2007). They are a subset of compliant mechanisms, in that they use the deflection of flexible members to achieve the desired motion. As a subset of compliant mechanisms, LEMs can provide feasible, repeatable solutions to advance the design and manufacturing of products. The advantages of LEMs are: reducing the number of parts, reducing cost, reducing weight, improving recyclability, increasing precision, and eliminating assembly (Jacobsen, 2007). The incorporation and use of LEMs offer many potential advantages in the design of mechanical products. These mechanisms can provide opportunities for more cost-effective, compact, easy to assemble, and modular products.

Orimimetic. Orimimetic means the ability to imitate folds. In an engineering design context, it refers to the ability to use the concept of folding to solve problems.

2.3 Principles From Graph Theory

Graph. A graph consists of points, called vertices, and connections, called edges (Marcus, 2008).

Planar graph. A graph with no edges crossing is a planar graph (Marcus, 2008).

Facet. The area enclosed by a planar graph is referred to as a facet.

Degree. The degree of a vertex is the number of edges that occur at that vertex (Marcus, 2008).

Degree sequence. The degree sequence of a graph is the degrees of each vertex listed in decreasing order (Marcus, 2008).

Regular graph, d-regular graph. A graph is considered regular if all its vertices are of the same degree. It can be referred to as a d-regular graph, where d us a non-negative integer where each vertex has degree d. (Marcus, 2008).

3 Origami Background

These sections describe the relevant history and current applications of origami to solve problems.

3.1 Origami History

The current origami movement began in the early 20th century, though historically origami has its beginning in several countries dating back to the Muromachi period, from 1333-1573 AD (Demaine and O'Rourke, 2007). Akira Yosihzawa and his origami works have been largely credited for the creative explosion in origami in the last century (Demaine and O'Rourke, 2007). Since the 1920's, origami design has become increasingly complex and varied. Initially origami was used to create figures and animals. As the art has progressed, the models have become increasingly complex, starting with a few to a few dozen steps to a few hundred or even a thousand steps. In the last twenty years new branches of origami have begun to be explored by more than artists; a growing number of mathematicians, educators, engineers and scientists have gathered to discuss the applications of origami in their respective fields (Demaine and O'Rourke, 2007).

Closely related to origami are other forms of paper engineering, such as kirigami, pop-up paper mechanisms, and origami architecture.

3.2 Origami Applications in Engineering and Science

Previous work has applied origami to map the transition between initial and final states, it has focused on studying the folding process. An example is airbag folding designs which unfold smoothly from their flat starting state to their final volume (Cromvik, 2007). The Diffractive Optics Group at Lawrence Livermore Laboratory is developing a telescope having a lens with a 100 meter diameter, but that could be collapsible enough to fit into a space vehicle having a 4 meter diameter and 10 meter length. Lang used origami to identify a design that would fit and maintain the integrity of the surface when deployed into space (Heller, 2003; Wu and You, 2010; Wei and Dai, 2009). Origami has also been applied to the crash box of a car to improve energy absorbtion in a low speed collision (Ma and You, 2010). Miura explored methods of folding maps that could be unfolded with the simple pull of a corner (Miura, 2002). In the 1980's he invented the Miura-ori pattern, which is used as a basis for folding solar arrays (Miura and Natori, 1985). In 2005, Mahadevan published findings that this same pattern exists in leaf folding, wings, and flower petals (Mahadevan and Rica, 2005).

4 Origami Mechanisms

This paper considers origami mechanisms as opposed to origami structures. The motion should exist after all folds are completed.

4.1 Kinematic Modelling of Origami Mechanisms

Origami mechanisms are modelled with hinge creases as joints and facets as links (Winder et al., 2009). Origami that is rigid-foldable is a "piecewise linear origami that is continuously transformable without the deformation of each facet" (Tachi, 2010). Rigid origami is also defined as having regions of the paper between crease lines that do not need to bend or twist in the folding process (i.e. the facet could be replaced with sheet metal and hinge creases replace by hinges and it would still fold up) (Hull, 1994; Tachi, 2006). Figure 3 shows an example of rigid origami by modelling the links of the square twist, a common origami unit for tessellations, with polypropylene sheet links, thus only allowing bending at the folds.

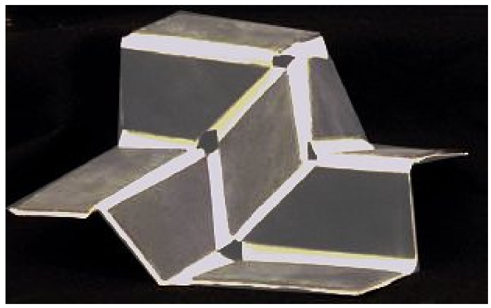

Fig. 3. A square twist origami out of polypropylene sheet and paper. The polypropylene shows that it behaves rigidly, where bending is only allowed at the folds.

4.2 Origami and Compliant Mechanisms

Origami mechanisms are compliant mechanisms; their motion is a result of the deflection of the material. As a material undergoes deformation, the resulting stored strain energy gives rise to an internal spring force. Balkcom notes that "the configuration of paper is determined by internal spring forces as well as external forces and constraints" (Balkcom and Mason, 2008). Thus, the origami can be viewed as a compliant mechanism which can be modelled using the pseudo-rigid-body model.

By focusing on origami in a fabricated state we only examine the folds that contribute to the mechanisms structure and motion, not the folds that were used to construct it.

4.2.1 Pseudo-Rigid-Body Model

The pseudo-rigid-body model (PRBM) uses rigid-body components, rigid links and springs, that have equal force-deflection characteristics to model the deflection of flexible members (Howell, 2001). Figure 4 shows the double slit and its corresponding PRBM.

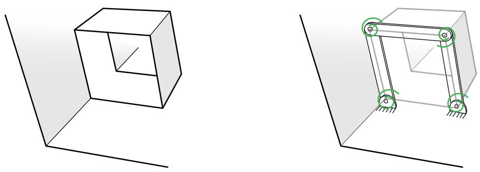

Fig. 4. A common double slit kirigami modelled and its corresponding PRBM.

4.3 Origami as a Lamina Emergent Mechanism

Because origami mechanisms are made from lamina materials it follows that they are LEMs. Traditionally, origami designs are judged by their efficiency and accuracy in terms of material usage and number of folds (Lang and Hull, 2005). For a model of an animal, efficiency is often defined by the ratio of the size of the final model to the initial sheet. This measure of efficiency can be especially useful in an orimimetic approach to LEM design.

Some branches of origami that are of special interest to LEMs are flat-folding origami, which is origami that folds to a flat state, and tessellations or tilings, which could be useful for LEM applications in arrays. The repetition of basic folds in tessellations leads to the development of an origami mechanism that expands and contracts thereby capturing the motion that would be required for an array-type structure.

Table 1 lists how origami has been applied in engineering. The potential for origami insights in LEM design can be seen by mapping existing applications of origami to the six application categories for LEMs suggested by Albrechtsen et al. (Albrechtsen et al., 2010).

Table 1. Origami Applications by LEM Technology Category

LEM Application Class	**Origami Corollaries**
Disposable mechanisms	Packaging
Novel Array mechanisms	Space sails Telescopes
Scalable mechanisms	Origami at nano-level Cellular origami
Surprising Motion mechanisms	Pop-up books
Shock-absorbing mechanisms	Crash box
Deployable mechanisms	Airbags Stents

5 Graph Theory Approach

Principles from graph theory can be used as a tool to understand how origami functions as a mechanism (Dobrjanskyj and Freudenstein, 1967). Graphs model origami and mechanisms on an abstract level and show their similarities. The abstraction of origami mechanisms to connected planar graphs demonstrates how the origami functions as a mechanism. Dai's carton folding research applies graph theory to show an equivalent mechanism (Dai and Jones, 2002). Origami mechanisms can be reduced to graphs which show how they behaves mechanically.

5.1 Modelling

Folds become line segments (edges) and links become nodes (vertices) in the graph. From such graphs it can be seen that origami mechanisms may be thought of as interconnected linkages, with each loop representing a linkage system. It is important to note that in graph theory the shape of an edge or the position of the vertices does not matter since graphs depict connections (Marcus, 2008).

The crease pattern can be related to the graph of the origami mechanism in some cases where all creases are hinge creases and the origami can be flattened via actuation. In this case the graph and crease pattern are considered dual graphs. Figure 5 shows the square twist with its crease pattern and corresponding graph overlaid. However, for the general case, the crease pattern may include structural and construction creases which aid in the folding of the origami but those creases do not contribute to the motion and therefore are not considered in the origami's corresponding graph.

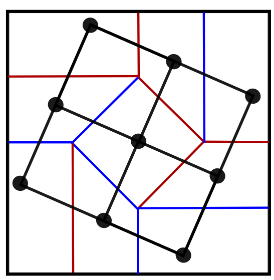

Fig. 5. The crease pattern for the square twist is shown with its corresponding graph overlaid in black. In the crease pattern the red corresponds to mountain folds and blue to valley folds.

Any simple planar connected graph with four segments connecting four vertices that is a 2-regular graph represents a four-bar mechanism. The graphs depict the degree of interconnection for each linkage system, this is shown in how each facet relates to its neighbouring facets. Each facet in a graph represents a linkage having the same number of links as nodes. For example, a facet (enclosed region) having four vertices (links) on the boundaries separating four segments (joints) is representative of a four-bar linkage.

6 Paper Mechanism Models

Four paper mechanisms are shown in Table 2 with their corresponding kinematic and graphical representations. For simplification, all four paper mechanisms are flat-folding mechanisms and are rigid-foldable. The first mechanism is a simple four-bar double-slit mechanism. The second is a mechanism with a series of 45-degree folds along a spine where links were cut from the same paper. Upon opening about the centre fold axis, the links rotate about an axis defined by the spine. The third is a series of square twists, which derive their name from the twisting motion of the central square when a corner is pulled to actuate the mechanism. Last is an example tessellation of the water bomb base fold.

The kinematic representations for each mechanism examine a portion of the larger mechanism and its motion. When the paper mechanisms have a flat initial state and they can be represented with kinematics, by definition they can be realized as a LEM. In addition, the graphs representing each paper mechanism show more abstractly the mechanism and how they are coupled. The graphs also indicate the type of fold, where mountain folds are shown in red and valley folds are in blue.

The four-bar double slit mechanism uses the PRBM to show how each crease can be modelled as a joint with a torsional spring. The four-bar double slit paper mechanism is a simple four-bar. Its graph is a 2-regular graph. It has three valley folds, and one mountain fold as indicated by the colours of edges in the graph. Each link has only two joints that define its motion.

For the 45-degree-fold twisting mechanism each centre fold, along the spine of the mechanism, is a 45 degree fold. The kinematic representation shows only four links along the spine. The 45-degree-fold twisting mechanism is a chain of four-bar mechanisms where those links that are not on the ends are share two links between each neighbouring linkage system, and each inside link is constrained by three joints with one of those joints shared between each neighbouring linkage system. Its graph has a degree sequence of (3,3,[20 more 3s],2,2,2,2).

The square twist mechanism is a series of coupled spherical mechanisms which collapse onto one another. Figure 3 shows the basic square twist, and the kinematic representation shows one of the four-bar linkages seen at the corner of the twisting square platform. It may be better to express the square twist as inter-connected four-bar spherical mechanisms, which may be seen in the graph representation. The square twist mechanism base graph is shown and has a degree sequence of (4,[6 more 4s],3,3,3,3,2,2,[6 more 2s]). The outer corner links each connect to only two joints. There are two neighbouring links on each end that are connected to three joints and are shared between two linkage systems. Of the 7 nodes of degree four, 3 are shared between four linkage systems and 4 are shared between three linkage systems.

Lastly, the kinematic representation for the water bomb base tessellation is shown. As a graph the water bomb base tessellation is a 3-regular graph. The water bomb base tessellation contains six-bar linkage systems that are interconnected. Each link is shared between three linkage systems via three joints. This is also the basic tessellation used in the origami stent design which allows for the mechanism to compact radially as well as lengthwise (Kuribayashi et al., 2006).

7 Conclusions

The ability to model origami and kirigami mechanisms with kinematics introduces a portion of the origami literature into the compliant mechanism field. It is anticipated that this will improve the ability to synthisise LEMs. In addition, by examining the graphs of some paper mechanisms it is understood that their motion is achieved because they are a system of coupled, or interconnected, mechanisms. The authors anticiplate that the abstraction to graph theory will provide direction for the development of synthesis techniques for LEMs.

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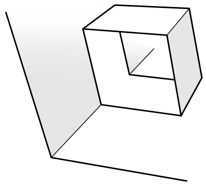
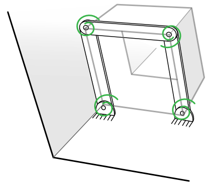
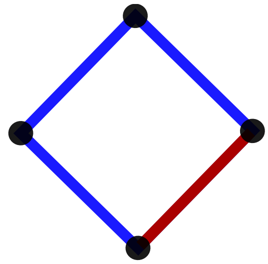
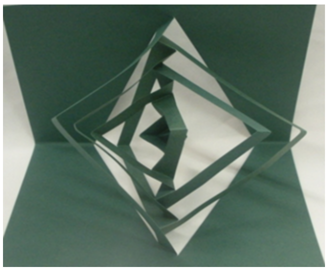
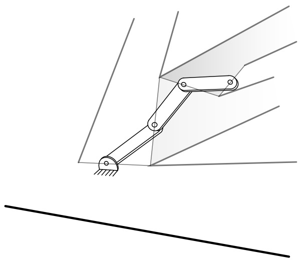
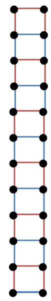
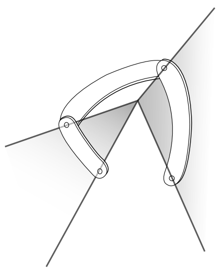
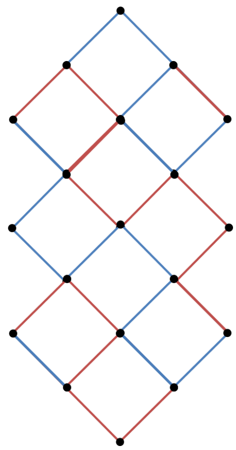
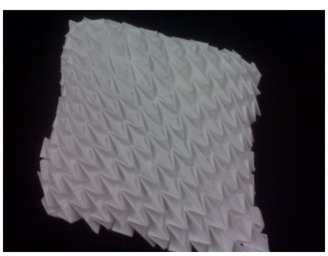
Table 2. Table of Mechanisms

Paper Mechanism	Kinematic Representation	Graph
 1. Four-Bar Double Slit		
 2. 45-degree-fold Twisting Mechanism		
 3. Square Twist		
 4. Water Bomb Base Tessellation